International Journal of Modern Physics B


# Second Generation of Composite Fermions and the Self-Similarity of the Fractional Quantum Hall Effect

M. O. Goerbig[1,2,*], P. Lederer[2], and C. Morais Smith[1,3]

[1] *Département de Physique, Université de Fribourg, Pérolles, CH-1700 Fribourg, Switzerland.*
[2] *Laboratoire de Physique des Solides, Bât. 510 (associé au CNRS), 91405 Orsay cedex, France.*
[3] *Institute for Theoretical Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands.*

Received 25 June 2004
Revised 20 August 2004

A recently developed model of interacting composite fermions, is used to investigate different composite-fermion phases. Their interaction potential allows for the formation of both solid and new quantum-liquid phases, which are interpreted in terms of second-generation composite fermions and which may be responsible for the fractional quantum Hall states observed at unusual filling factors, such as $\nu = 4/11$. Projection of the composite-fermion dynamics to a single level, involved in the derivation of the Hamiltonian of interacting composite fermions, reveals the underlying self-similarity of the model.

*Keywords*: quantum Hall effect; fermions in reduced dimensions

## 1. Introduction

Most fractional quantum Hall effects (FQHE) may be understood as an integer quantum Hall effect (IQHE) of a quasi-particle, a so-called composite fermion (CF), which consists of an electron and a vortex-like object with vorticity $2s$.[1] The formation of CFs is due to the presence of strong correlations between the interacting electrons in a partially filled Landau level (LL), the filling of which is characterized by the ratio $\nu = n_{el}/n_B$ between the electronic and the flux densities, $n_{el}$ and $n_B = B/(h/e)$, respectively. Because of its fractional charge, the CF experiences a reduced coupling $(eB)^* = eB/(2sp+1)$ to the external magnetic field $B$ and, in an approximate sense, forms LLs (CF-LLs) itself.[2] The FQHE arises when the CF filling factor $\nu^* = hn_{el}/(eB)^*$ is an integer $p$. Because the electronic and CF filling factors are related by $\nu = \nu^*/(2s\nu^* + 1)$, this type of FQHE is expected for the series $\nu = p/(2sp + 1)$.

Recently, Pan *et al.* have observed a FQHE at $\nu = 4/11$, which may not be described as an IQHE of CFs, but should rather be viewed as a FQHE of CFs at $\nu^* =$

*E-mail: goerbig@lps.u-psud.fr

$1 + 1/3$.[3] This discovery has renewed the interest in a possible self-similarity of the FQHE, investigated by Mani and v. Klitzing on the basis of scaling transformations.[4] It is natural to interpret such new FQHE states in terms of higher-generation CFs: the $4/11$ state may thus be viewed as an IQHE of second-generation CFs ($C^2$Fs).[5,6,7] However, recent numerical investigations by Wójs *et al.* indicate that the interaction potential between quasi-particles in $1/3 < \nu < 2/5$ is not sufficiently short-range to allow for the formation of higher-generation CFs.[8] Furtheremore, numerical-diagonalization studies in the CF basis reveal an alternation between compressible and incompressible states at $\nu = 4/11$ with varying CF number and have not provided a conclusive answer about the nature of this state in the thermodynamic limit.[9]

Here, we investigate the self-similarity of the FQHE within a model of interacting spin-polarized composite fermions at $\nu^* \neq p$, which we have recently derived in the framework of Murthy and Shankar's Hamiltonian theory.[2] The self-similarity is found in the mathematical structure of the model: its Hamiltonian has the same form as that of electrons restricted to a single LL if one replaces the electronic by the CF interaction potential. Furthermore, the CF density operators, restricted to a single CF-LL, satisfy the same commutation relations as the projected operators of the electron density, in terms of a renormalized magnetic length $l_B^* = \sqrt{\hbar/(eB)^*}$.[6] However, the CF interaction potential, which has been derived within the model, and that for electrons in a single LL have a *different* form and cannot be related to each other by simple rescaling of the magnetic length. The existence of higher-generation CFs is therefore not guaranteed by the self-similarity of the model. Nevertheless, detailed energy calculations for competing CF quantum-liquid and solid phases, such as bubbles and stripes of CFs, have been performed within the model and indicate the stability of some $C^2$F states.[10]

## 2. Model of Interacting Composite Fermions

At $p < \nu^* < p + 1$, one is confronted with a ground-state degeneracy in a model of non-interacting CFs. This degeneracy is lifted by the residual CF interactions, which may be taken into account in the Hamiltonian[6] ($l_B = \sqrt{\hbar/eB} \equiv 1$)

$$\hat{H}(s,p) = \frac{1}{2A}\sum_{\mathbf{q}} v_{s,p}^{CF}(q)\bar{\rho}^{CF}(-\mathbf{q})\bar{\rho}^{CF}(\mathbf{q}), \tag{1}$$

with the CF-interaction potential, given in terms of Laguerre polynomials $L_p(x)$,

$$v_{s,p}^{CF}(q) = \frac{2\pi e^2}{\epsilon(q)q} e^{-q^2 l_B^{*2}/2}\left[L_p\left(\frac{q^2 l_B^{*2} c^2}{2}\right) - c^2 e^{-q^2/2c^2} L_p\left(\frac{q^2 l_B^{*2}}{2c^2}\right)\right]^2 \tag{2}$$

and the vortex charge $c^2 = 2ps/(2ps+1)$.[6] This model describes low-energy excitations within the same CF-LL, and the restricted CF density operators $\bar{\rho}^{CF}(-\mathbf{q})$ satisfy the commutation relations $[\bar{\rho}^{CF}(\mathbf{q}), \bar{\rho}^{CF}(\mathbf{k})] = 2i\sin\left[(\mathbf{q}\times\mathbf{k})_z l_B^{*2}/2\right]\bar{\rho}^{CF}(\mathbf{q}+\mathbf{k})$. In contrast to prior investigations, where the interaction potential has been constructed from a few numerically determined pseudopotentials,[11,8] here, it has been

derived in a set of transformations,[6] in the framework of the Hamiltonian theory of the FQHE.[2] Inter-CF-LL excitations are taken into account with the help of a $q$-dependent dielectric function $\epsilon(q)$ in the CF interaction potential (2).[6]

Because of its similarity with the electronic model, the Hamiltonian (1) may be analyzed by the same techniques as the one of electrons restricted to a single level.[10] The energy $E_{coh}^{L}(s,p;\tilde{s})$ of the quantum-liquid phases at $\bar{\nu}^* = 1/(2\tilde{s}+1)$, where $\tilde{s}$ is an integer, and $\bar{\nu}^* = \nu^* - p$ is the partial filling of the $p$-th CF-LL, may be calculated in Laughlin's wave function approach.[12] Note that we restrict the discussion to Laughlin liquids here although other incompressible liquids may also play a role at certain CF filling factors. At $\bar{\nu}^* \neq 1/(2\tilde{s}+1)$, the energy $\Delta_{s,p}^{qp/qh}(\tilde{s})$ of the excited quasiparticles [for $\bar{\nu}^* > 1/(2\tilde{s}+1)$] or quasiholes [for $\bar{\nu}^* < 1/(2\tilde{s}+1)$] has to be taken into account. The latter may be interpreted as $C^2$Fs or $C^2$F-holes excited to a higher level, which raise the energy of the quantum-liquid phases away from $\bar{\nu}^* = 1/(2\tilde{s}+1)$, where local minima in form of cusps are obtained. They are at the origin of the incompressibility of the quantum liquids. The energy of CF-solid phases may be calculated in the Hartree-Fock approximation, which has been used in the study of electron-solid phases in higher LLs.[13,14,15] The expressions for the energies of the quantum-liquid and CF solid phases may be found in Ref. 10.

## 3. Results

The results for the energies of the different CF phases are shown in the Fig.1. In contrast to Ref. 10, the energies are calculated with a screened interaction potential. The quantum-liquid ($C^2$F) phases are stable around $\nu^* = 1+1/3$ and $1+1/5$, which correspond to the electronic fillings $\nu = 4/11$ and $6/17$, respectively. Whereas a spin-polarized FQHE at $\nu = 4/11$ has been observed by Pan *et al.*, only a tiny local minimum in the longitudinal resistance hints to the existence of a possible $6/17$ state.[3] Indeed, a $C^2$F state is close in energy to a CF Wigner crystal ($M = 1$), the energy of which is lowered if one takes into account an underlying impurity potential: a solid phase may take advantage of the minima of such a potential by deformation of its crystalline structure, in analogy with the electronic case.[15] It is therefore not clear whether the quantum liquid at $\nu = 6/17$ survives in the presence of impurities. At half-filling, one finds a CF stripe phase, which may give rise to an anisotropic longitudinal resistance, which has recently been observed at $\nu = 13/8$.[16] While these results are similar to those obtained for the competing *electronic* phases in the first excited LL ($n = 1$),[15] as one may expect from self-similarity arguments, there are differences due to the different form of the CF interaction potential. In $p = 1$, the $1/3$ state, *e.g.*, is more stable than the $1/5$ state in the CF model, and a two-CF bubble phase ($M = 1$) is unstable (see figure), in contrast to the $n = 1$-LL.[15]

Note that the CF Wigner crystal has a lower energy than the quantum liquid above $\nu \simeq 0.44$. This transition point is shifted to lower densities if one takes into account impurity effects and the repulsive interaction between the excited $C^2$Fs, which leads to non-linear slopes in the quantum-liquid energies at $\bar{\nu}^* \neq 1/(2\tilde{s}+1)$.

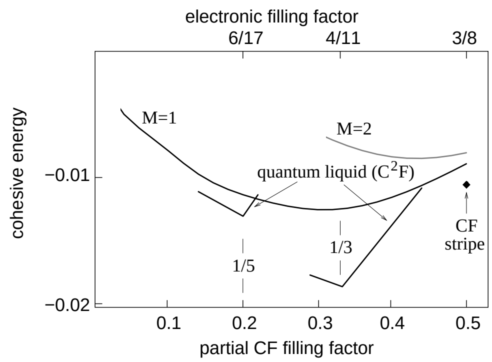


Fig. 1. Cohesive energies of the different CF phases, in units of $e^2/\epsilon l_B$.

Impurities favor the CF Wigner crystal also at small values of $\bar{\nu}^*$. Therefore, the C$^2$F phases are surrounded by insulating CF Wigner crystals, and this may lead to a *reentrance phenomenon of the FQHE*,[10] in analogy with the reentrance of the IQHE in the first excited LL, observed by Eisenstein *et al.*[17]

## Acknowledgements

We acknowledge fruitful discussions with M. P. A. Fisher, R. Mani, R. Moessner, R. Morf, and A. Wójs. This work was supported by the Swiss National Foundation for Scientific Research under grant No. 620-62868.00.

## References

1. J. K. Jain, *Phys. Rev. Lett.* **63**, 199 (1989).
2. G. Murthy and R. Shankar, *Rev. Mod. Phys.* **75**, 1101 (2003).
3. W. Pan *et al.*, *Phys. Rev. Lett.* **90**, 016801 (2003).
4. R. G. Mani and K. v. Klitzing, *Z. Phys. B* **100**, 635 (1996).
5. J. H. Smet, *Nature* **422**, 391 (2003).
6. M. O. Goerbig, P. Lederer, and C. Morais Smith, *Phys. Rev.* **B69**, 155324 (2004); preprint: cond-mat/0401340.
7. A. Lopez and E. Fradkin, *Phys. Rev.* **B69**, 155322 (2004).
8. A. Wójs, K.-S Yi, and J. Quinn, *Phys. Rev.* **B69**, 205322 (2004).
9. S. S. Mandal and J. K. Jain, *Phys. Rev.* **B66**, 155302 (2002); C.-C. Chang and J. K. Jain, *Phys. Rev. Lett.* **92**, 196806 (2004).
10. M. O. Goerbig, P. Lederer, and C. Morais Smith, preprint: cond-mat/0403016.
11. S.-Y. Lee *et al.*, *Phys. Rev. Lett.* **87**, 256803 (2001); *Phys. Rev.* **B66**, 085336 (2002).
12. R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).
13. M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, *Phys. Rev.* **B54**, 1853 (1996).
14. R. Moessner and J. T. Chalker, *Phys. Rev.* **B54**, 5006 (1996).
15. M. O. Goerbig, P. Lederer, and C. Morais Smith, *Phys. Rev.* **B69**, 115327 (2004).
16. F. Fischer *et al.*, *Physica* **E22**, 108 (2004); $\nu = 13/8$ in the upper spin-branch of the lowest LL is the particle-hole symmetric filling of $\nu = 3/8$.
17. J. P. Eisenstein *et al.*, *Phys. Rev. Lett.* **88**, 076801 (2002).